Axion cosmology with $N_{DW} > 1$

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Assumptions in cosmology

The “vanilla” scenario based on three assumptions:

1. Peccei-Quinn symmetry has been broken after inflation.

2. Standard expansion history (i.e. radiation domination) after axion number is fixed.

3. Cosmic strings and domain walls decay soon after the QCD phase transition ($N_{\rm DW} = 1$).

Assumptions in cosmology

The “vanilla” scenario based on three assumptions:

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3. Cosmic strings and domain walls decay soon after the QCD phase transition $(N_{\rm DW} = 1)$

What if it is not the case?

Vacuum structure after the QCD phase transition

- The axion-gluon coupling comes from the color anomaly.

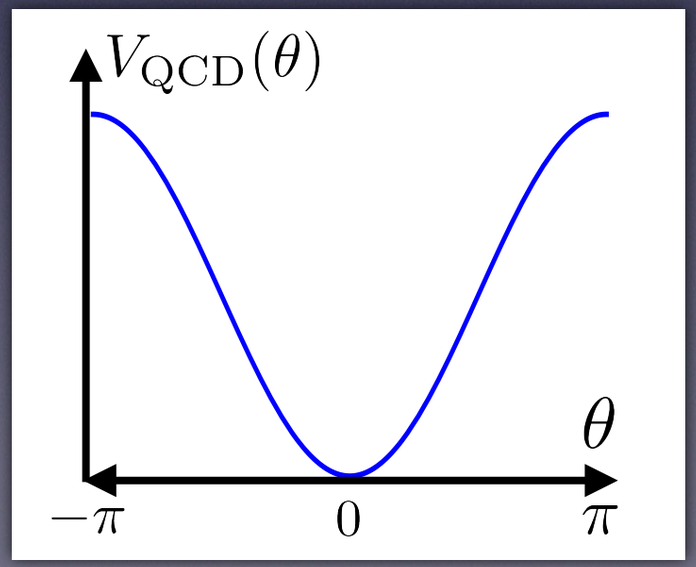
$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi}\mathcal{N}\frac{a}{v_{\mathrm{PQ}}}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$\mathcal{N}$: color anomaly coefficient

v_{PQ} : Peccei-Quinn symmetry breaking scale

- $\mathcal{N} \neq 0$ is required to solve the strong CP problem.
- Axion potential is 2π periodic in $\theta = \mathcal{N}a/v_{\mathrm{PQ}}$.

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi}\theta G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \Rightarrow V_{\mathrm{QCD}}(\theta)$$



Vacuum structure after the QCD phase transition

- Axion is a periodic angular field defined in the range

$$a/v_{\mathrm{PQ}} \in (-\pi, \pi).$$

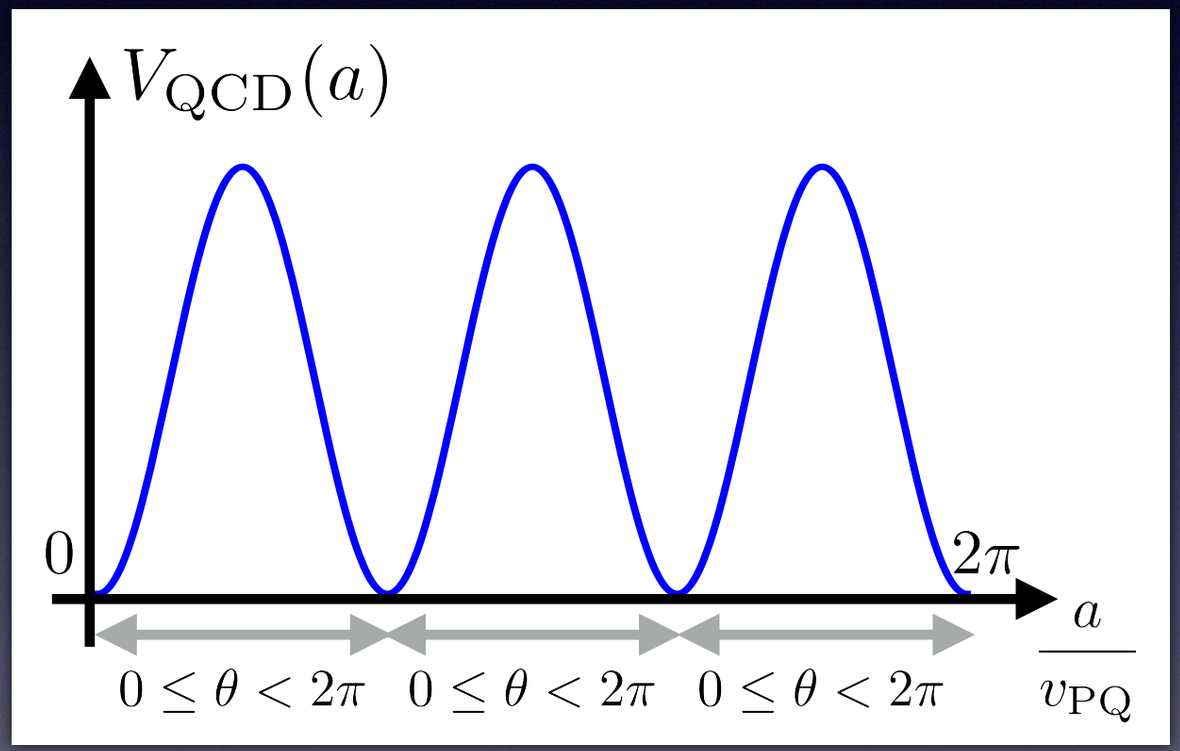
- Difference in periodicity:

  When θ changes by 2π,

  a/v_{PQ} changes by $2\pi/\mathcal{N}$.

  $(\theta = \mathcal{N} a/v_{\mathrm{PQ}})$

  ➡ $|\mathcal{N}|$ degenerate minima

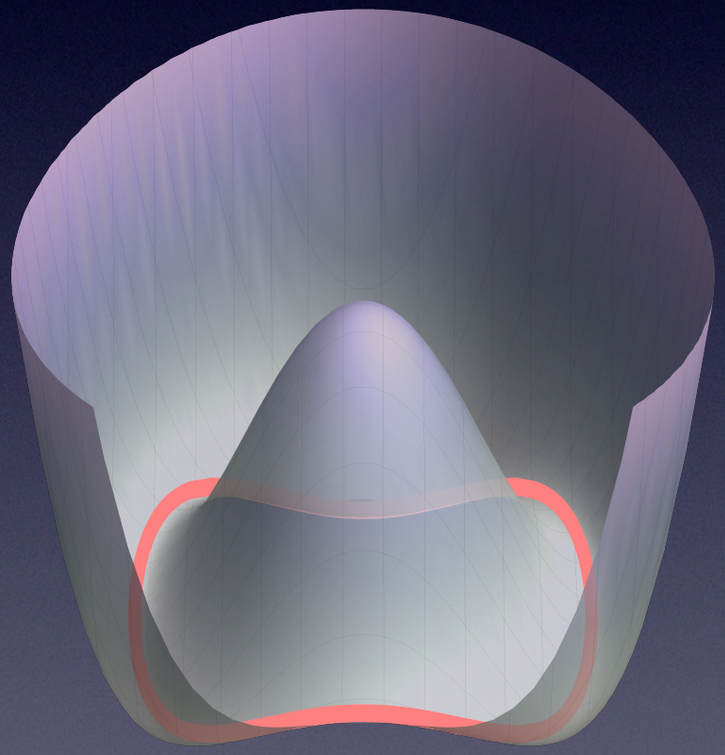


- In this context, the color anomaly coefficient is called the “domain wall number” ($N_{\mathrm{DW}} = |\mathcal{N}|$).

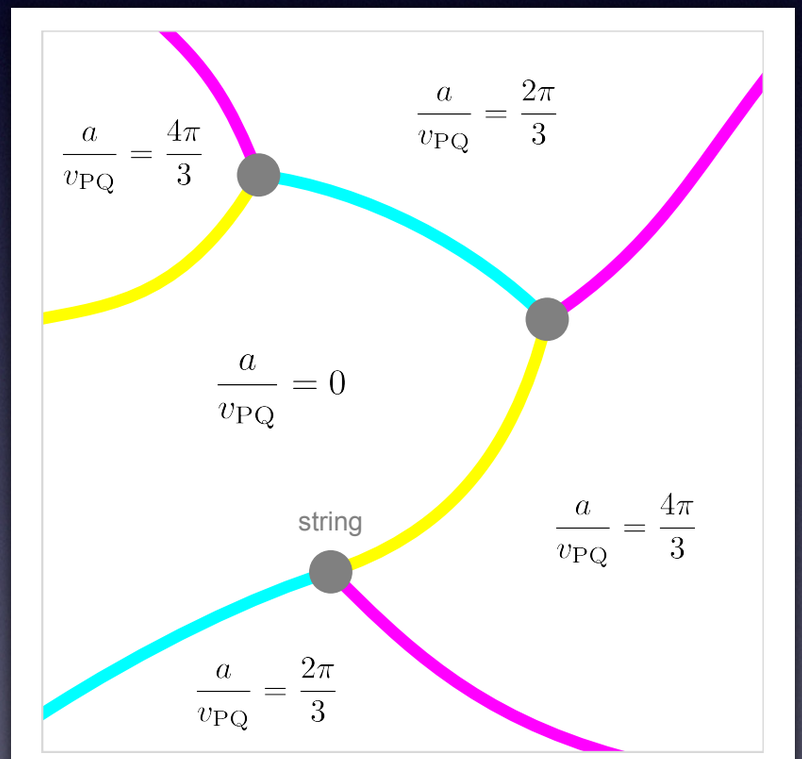
Axionic domain walls

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi), \quad V(\phi) = \lambda \left(|\phi|^2 - \frac{v_{\rm PQ}^2}{2} \right)^2 + \chi(T) \left[1 - \cos\left(N_{\rm DW} \frac{a}{v_{\rm PQ}} \right) \right]$$

$V(\phi)$

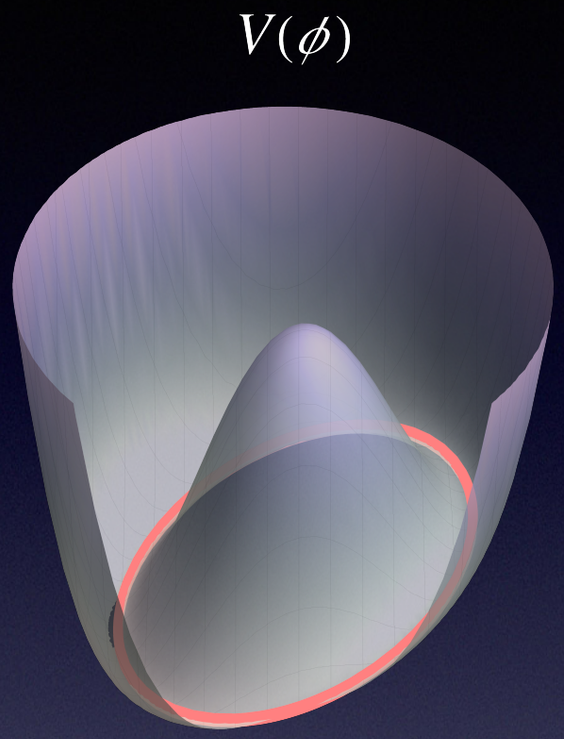


Position space

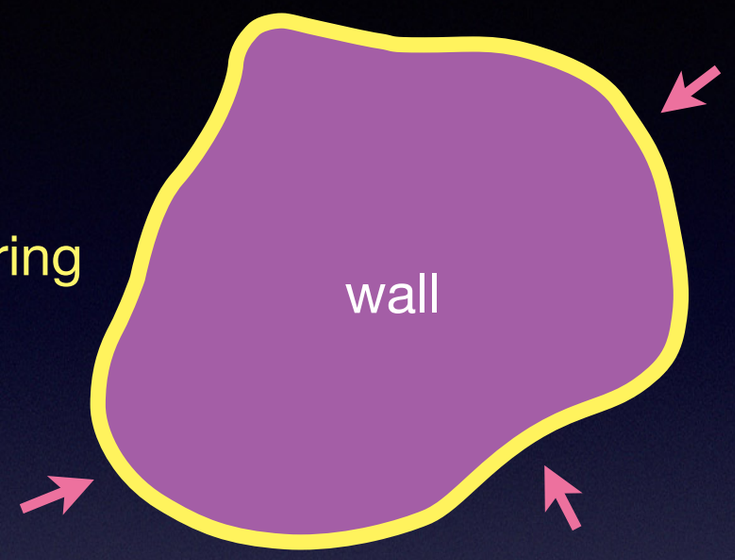


Low energy effective potential may have N_{DW} degenerate minima.
Domain walls form around the epoch of the QCD phase transition.

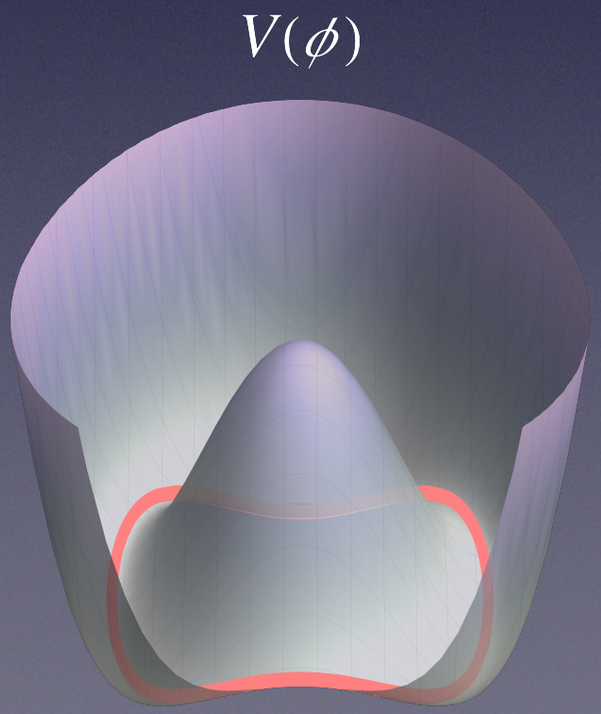
$N_{DW} = 1$ vs. $N_{DW} > 1$



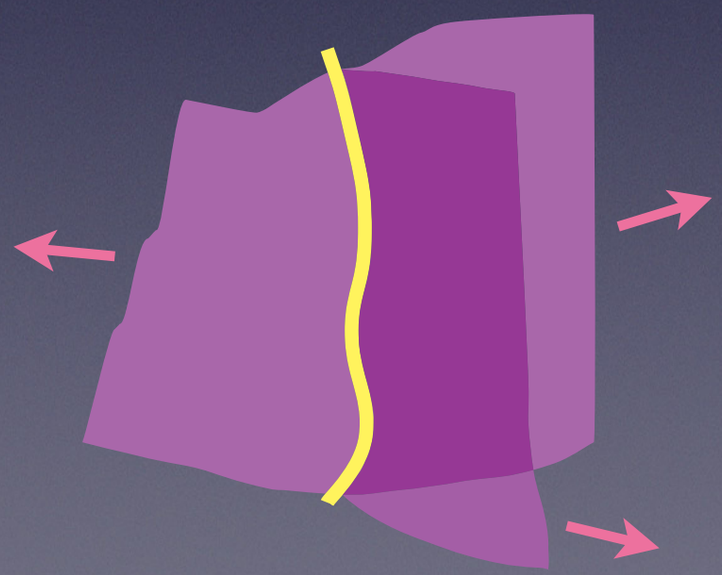
$N_{DW} = 1$



unstable, going to collapse



$N_{DW} = 3$



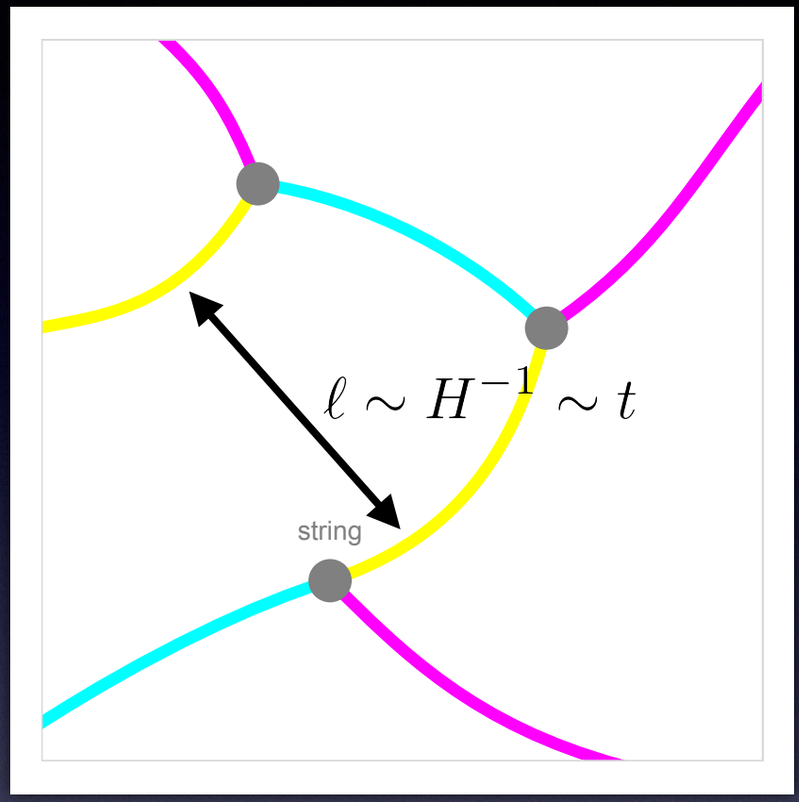
stable!

Domain wall problem [Sikivie (1982)]

- Numerical studies imply that the evolution of domain walls can be described by the *scaling solution*:

  $\mathcal{O}(1)$ walls per Hubble radius.

  [Press, Ryden and Spergel (1989)]



- Energy density

$$\rho_{\rm wall} \sim \left.\frac{\sigma\ell^2}{\ell^3}\right|_{\ell\sim t} \sim \frac{\sigma}{t}$$

$\sigma = 8 m_a f_a^2$
surface energy density

decays slower than $\rho_{\rm matter} \propto R^{-3}$ and $\rho_{\rm radiation} \propto R^{-4}$.

- Walls come to overclose the universe, conflicting with the standard cosmology (domain wall problem).

Theory prediction for N_{DW}

- N_{DW} is related to color anomaly coefficient.

$$N_{\rm DW} = \left| 2 \sum_{i=L} {\rm Tr} Q_{\rm PQ}(q_i) T_a^2(q_i) - 2 \sum_{i=R} {\rm Tr} Q_{\rm PQ}(q_i) T_a^2(q_i) \right|$$

$Q_{\rm PQ}(q_i)$: $U(1)_{PQ}$ charge for quark species

T_a : generators of $SU(3)_c$

- KSVZ model predicts $N_{DW} = 1$, [Kim (1979), Shifman, Vainshtein and Zakharov (1980)]
  if there is only one heavy exotic quark with some specific representation of the SM gauge group.

- DFSZ model predicts $N_{DW} = 6$. [Zhitnitsky (1980), Dine, Fischler and Srednicki (1981)]
  (no exotic quark, all the SM quarks are charged under $U(1)_{PQ}$.)

Theory prediction for N_{DW}

- N_{DW} is rela

$$N_{\rm DW} = \left| \frac{}{} \right| \qquad q_i) T_a^2(q_i) \Big|$$

$$Q_{\rm PQ}$$

- KSVZ mod
  if there is
  representa

- DFSZ mod
  (no exotic

…but there are a lot of variations of the KSVZ model!

[Di Luzio, Mescia and Nardi (2017)]

TABLE II. R_Q allowing for $d \leq 4$ and $d = 5$ decay operators ($\sigma G \equiv \sigma_{\mu\nu} G^{\mu\nu}$) and yielding a LP above 10^{18} GeV. The anomaly contribution to $g_{a\gamma\gamma}$ is given in the fourth column, and the DW number in the fifth one.

R_Q	O_{Qq}	$\Lambda_{\rm LP}^{R_Q}$ (GeV)	E/N	$N_{\rm DW}$
R_1: $(3,1,-\frac{1}{3})$	$\bar{Q}_L d_R$	$9.3 \times 10^{38} (g_1)$	2/3	1
R_2: $(3,1,+\frac{2}{3})$	$\bar{Q}_L u_R$	$5.4 \times 10^{34} (g_1)$	8/3	1
R_3: $(3,2,+\frac{1}{6})$	$\bar{Q}_R q_L$	$6.5 \times 10^{39} (g_1)$	5/3	2
R_4: $(3,2,-\frac{5}{6})$	$\bar{Q}_L d_R H^\dagger$	$4.3 \times 10^{27} (g_1)$	17/3	2
R_5: $(3,2,+\frac{7}{6})$	$\bar{Q}_L u_R H$	$5.6 \times 10^{22} (g_1)$	29/3	2
R_6: $(3,3,-\frac{1}{3})$	$\bar{Q}_R q_L H^\dagger$	$5.1 \times 10^{30} (g_2)$	14/3	3
R_7: $(3,3,+\frac{2}{3})$	$\bar{Q}_R q_L H$	$6.6 \times 10^{27} (g_2)$	20/3	3
R_8: $(3,3,-\frac{4}{3})$	$\bar{Q}_L d_R H^{\dagger 2}$	$3.5 \times 10^{18} (g_1)$	44/3	3
R_9: $(\bar{6},1,-\frac{1}{3})$	$\bar{Q}_L \sigma d_R G$	$2.3 \times 10^{37} (g_1)$	4/15	5
R_{10}: $(\bar{6},1,+\frac{2}{3})$	$\bar{Q}_L \sigma u_R G$	$5.1 \times 10^{30} (g_1)$	16/15	5
R_{11}: $(\bar{6},2,+\frac{1}{6})$	$\bar{Q}_R \sigma q_L G$	$7.3 \times 10^{38} (g_1)$	2/3	10
R_{12}:$(8,1,-1)$	$\bar{Q}_L \sigma e_R G$	$7.6 \times 10^{22} (g_1)$	8/3	6
R_{13}: $(8,2,-\frac{1}{2})$	$\bar{Q}_R \sigma \ell_L G$	$6.7 \times 10^{27} (g_1)$	4/3	12
R_{14}: $(15,1,-\frac{1}{3})$	$\bar{Q}_L \sigma d_R G$	$8.3 \times 10^{21} (g_3)$	1/6	20
R_{15}: $(15,1,+\frac{2}{3})$	$\bar{Q}_L \sigma u_R G$	$7.6 \times 10^{21} (g_3)$	2/3	20

in and Zakharov (1980)]

specific

er and Srednicki (1981)]

under $U(1)_{PQ}$.)

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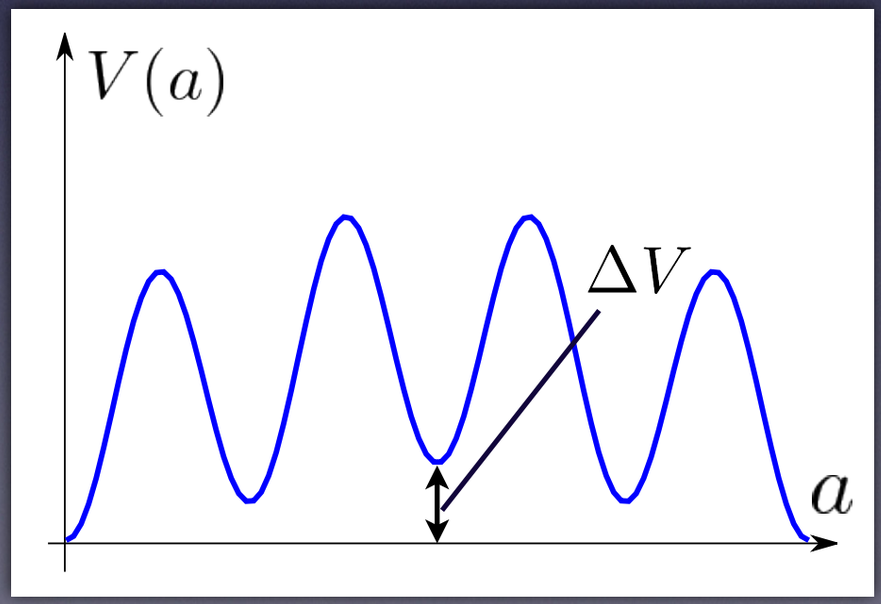
Avoiding the domain wall problem

- Option 1: Inflation after the PQ phase transition
  - Strings and domain walls are diluted away.
  - reduced to the pre-inflationary scenario…
- Option 2: Destabilizing domain walls [Sikivie (1982)]
  - Assume there exists an additional term which explicitly breaks the PQ symmetry and lifts degenerate vacua (energy bias).
  - The breaking term leads to the late-time annihilation of domain walls.

(The origin of the bias term will be discussed shortly.)

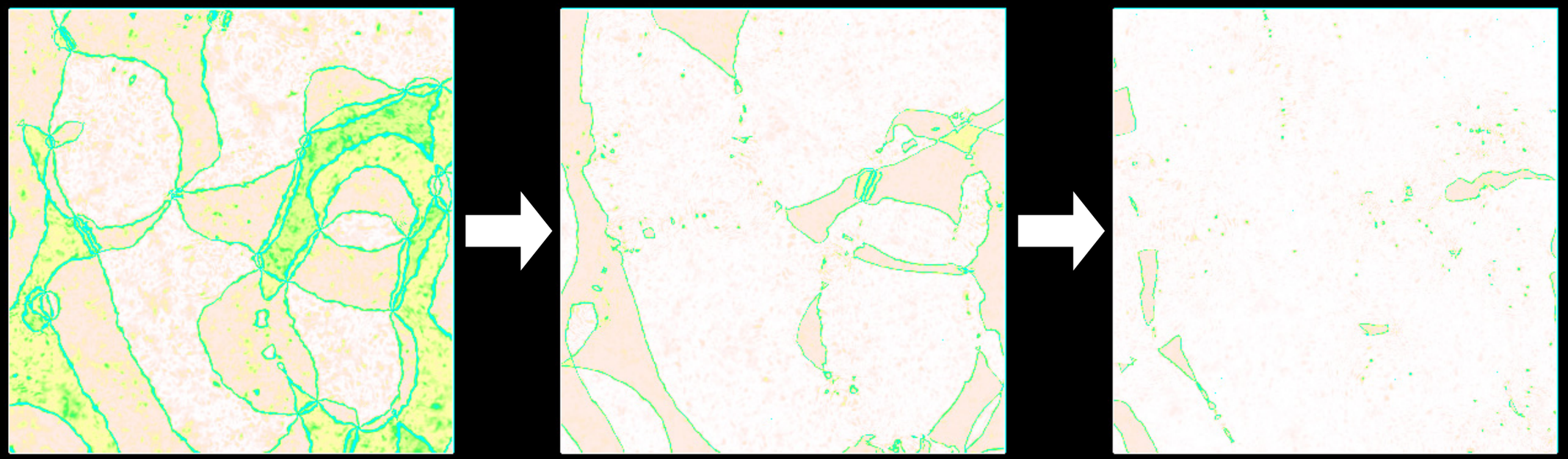


Annihilation of domain walls

- The bias term acts as a pressure force $p_V \sim \Delta V$ on the wall.
- Annihilation occurs when the tension $p_T \sim \sigma/t$ becomes comparable with the pressure:

$$t_{\rm dec} \sim \sigma/\Delta V \sim m_a f_a^2/\Delta V$$

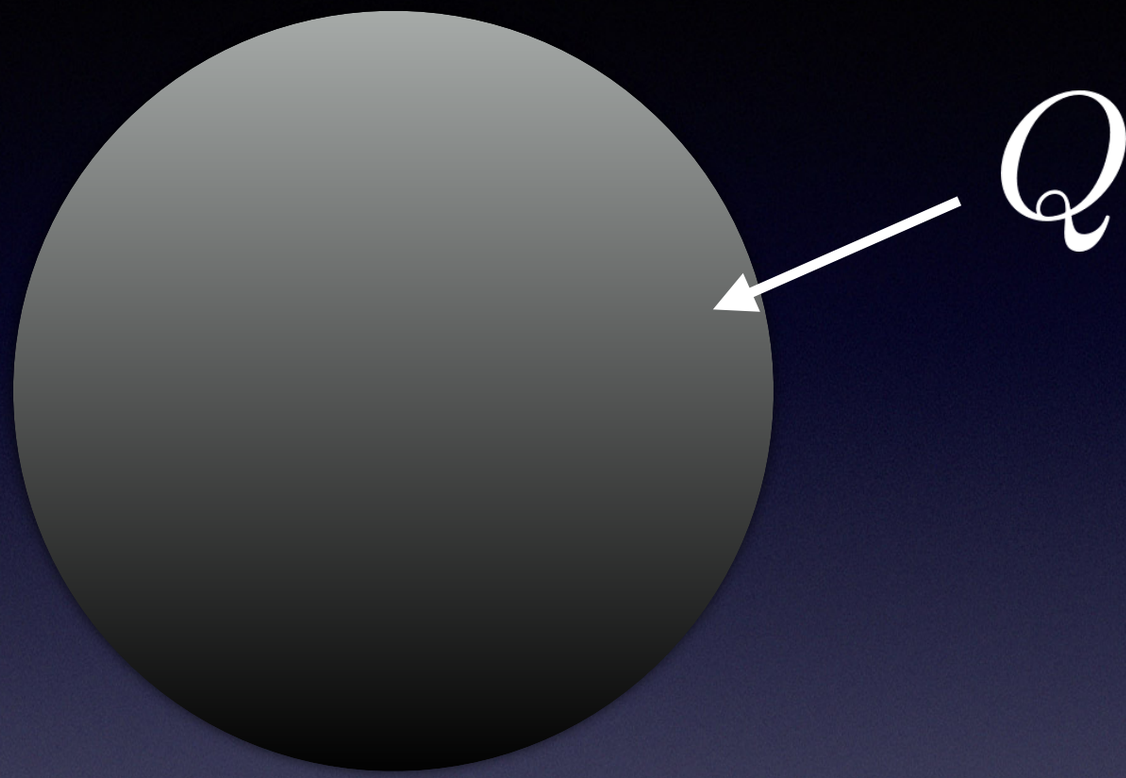
$$\Rightarrow \quad T_{\rm dec} \sim \mathcal{O}(10)\,{\rm MeV} \left(\frac{\Delta V/f_a^4}{10^{-48}}\right)^{1/2} \left(\frac{f_a}{10^9\,{\rm GeV}}\right)^{3/2}$$



$N_{\rm DW} = 6,\ \Delta V/v_{\rm PQ}^4 = 1.2 \times 10^{-4}$

[Kawasaki, KS and Sekiguchi (2015)]

Origin of the bias term?



$U(1)_{PQ}$ may not be an exact symmetry:

Any global symmetry can be spoiled by gravitational effects.

(No chance to keep track of the global charge thrown into black hole.)

How large is the breaking effect?

Higher dimensional operators

[Holman et al., Kamionkowski and March-Russell,
Barr and Seckel, Ghigna, Lusignoli and Roncadelli, Dine (1992)]

$$\mathcal{L} \supset \frac{g}{M_{\rm Pl}^{N-4}}\phi^N + \text{h.c.} \quad \Rightarrow \quad \frac{\Delta V}{f_a^4} = -\frac{|g| N_{\rm DW}^N}{(\sqrt{2})^{N-2}} \left(\frac{f_a}{M_{\rm Pl}}\right)^{N-4} \cos\left(N\frac{a}{v_{\rm PQ}} + \delta\right)$$

induces CP violation effects,

$$\Delta\theta \simeq \frac{N|g|N_{\rm DW}^{N-1}}{(\sqrt{2})^{N-2}} \left(\frac{f_a}{M_{\rm Pl}}\right)^{N-2} \frac{M_{\rm Pl}^2}{m_a^2} \sin\delta$$

A threat for the axion solution to the strong CP problem.

(e.g. For N = 5, $|g| \lesssim 10^{-60}$ is required in order to avoid $\Delta\theta > 10^{-10}$.)

Possible solution:

Assume that the PQ symmetry is an accidental symmetry descending from an exact (discrete) gauge symmetry.

[Choi, Nilles, Ramos-Sanchez and Vaudrevange (2009)]
[Fukuda, Ibe, Suzuki and Yanagida (2017)]

Higher dim. operators with N = 9 or 10 may work well.

[Ringwald and KS (2016)]

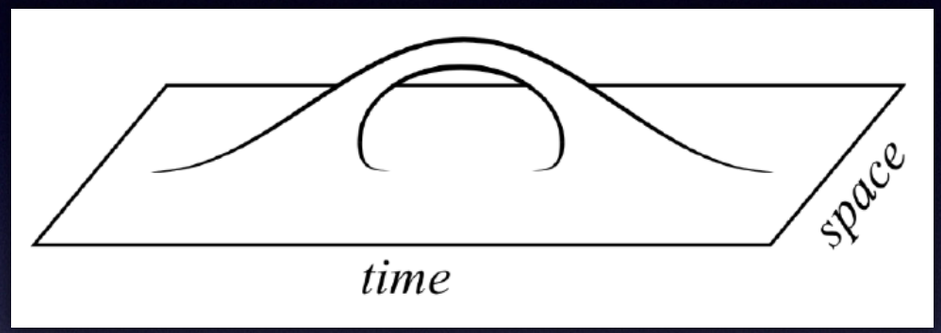
Non-perturbative gravity effects

[Kallosh, Linde, Linde and Susskind (1995)]
[Alonso and Urbano (2017)]

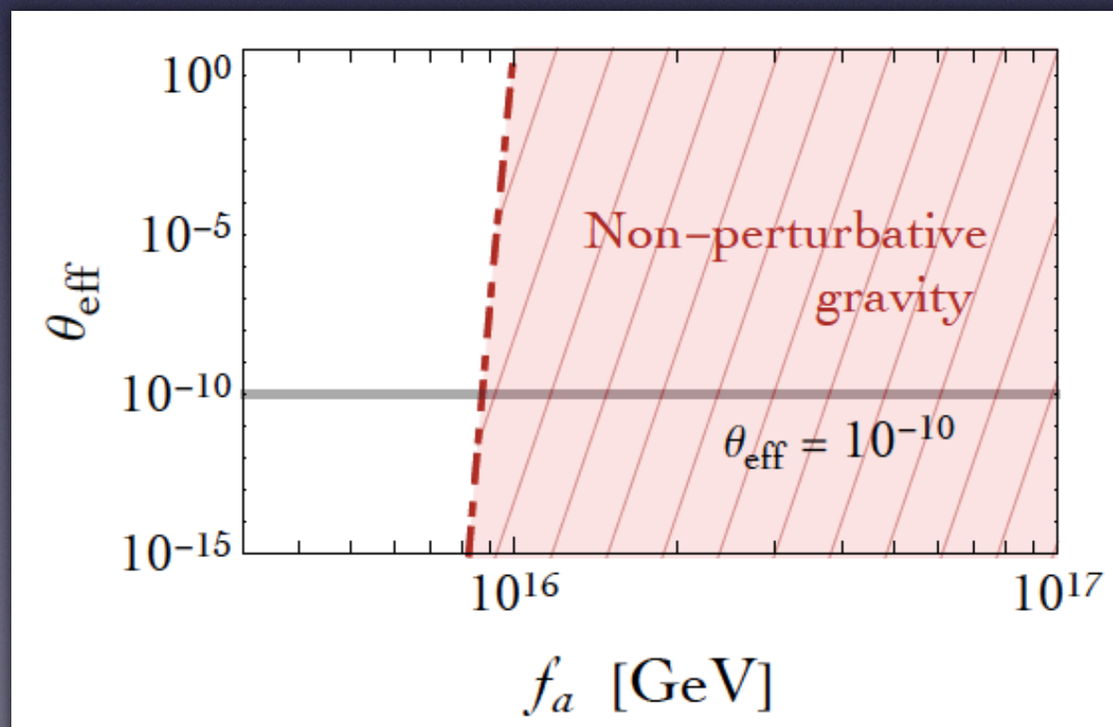
- Gravitational instantons (“wormholes”)

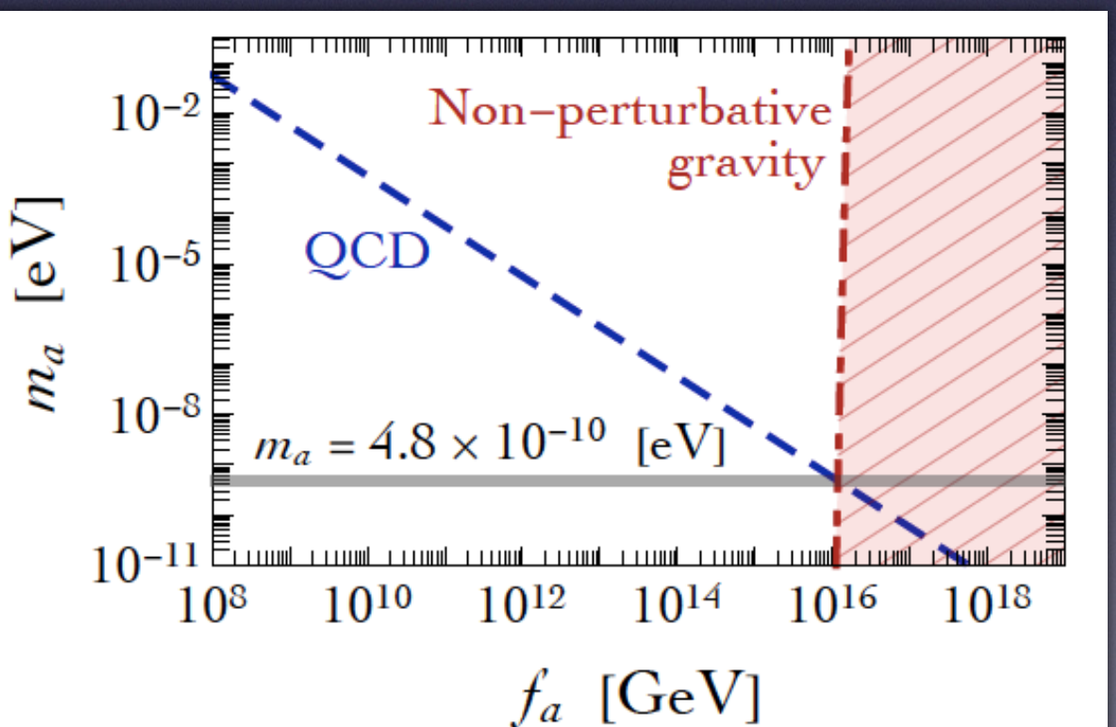
$$\Delta V \sim M_{\rm Pl}^2 f_a^2 e^{-S_{\rm inst}} \cos\left(\frac{a}{f_a} - \delta\right)$$

$$S_{\rm inst} = \frac{\sqrt{3\pi} M_{\rm Pl}}{8 f_a}\left(1 - \frac{2}{\pi}\right)$$



- Effects are exponentially small?





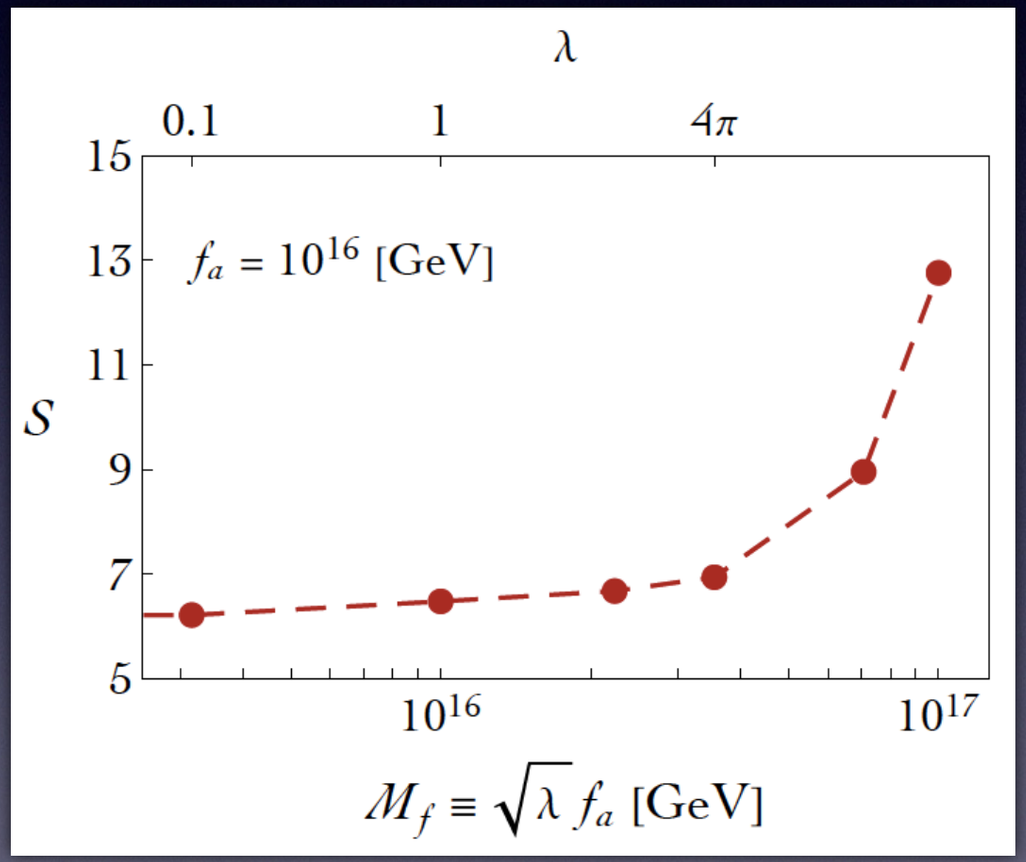
Non-perturbative gravity effects

[Kallosh, Linde, Linde and Susskind (1995)]
[Alonso and Urbano (2017)]

- The previous result is derived based on the setup of a Goldstone field minimally coupled to Einstein gravity.

- Including dynamical radial mode or modification of gravity can significantly alter the result.

- The lack of the UV complete theory of gravity makes any argument highly speculative.



- However, we can take a phenomenological approach to constrain these effects.

Phenomenological approach

- CP violation

  The explicit symmetry breaking term shifts the minimum of the potential and spoils the original Peccei-Quinn solution to the strong CP problem.

$$\Delta V = |\Delta V| \cos\left(\frac{a}{v_{\rm PQ}} - \delta\right)$$

$$\frac{\langle a \rangle}{f_a} = \frac{\frac{f_a^2}{N_{\rm DW}} \frac{|\Delta V|}{f_a^4} \sin\delta}{m_a^2 + \frac{f_a^2}{N_{\rm DW}^2} \frac{|\Delta V|}{f_a^4} \cos\delta} < \mathcal{O}(10^{-10})$$

➡ Upper limit on $|\Delta V|$

- Dark matter abundance

  Long-lived domain walls may produce too much cold axions.

$$\Omega_a h^2(f_a, \Delta V) < 0.12$$

➡ Lower limit on $|\Delta V|$

Simulation of $N_{DW} > 1$ domain walls

[Hiramatsu, Kawasaki, KS and Sekiguchi (2013)]
[Kawasaki, KS and Sekiguchi (2015)]

Solve the EOM for the complex scalar,
including the QCD potential with N_{DW} minima (and bias term).

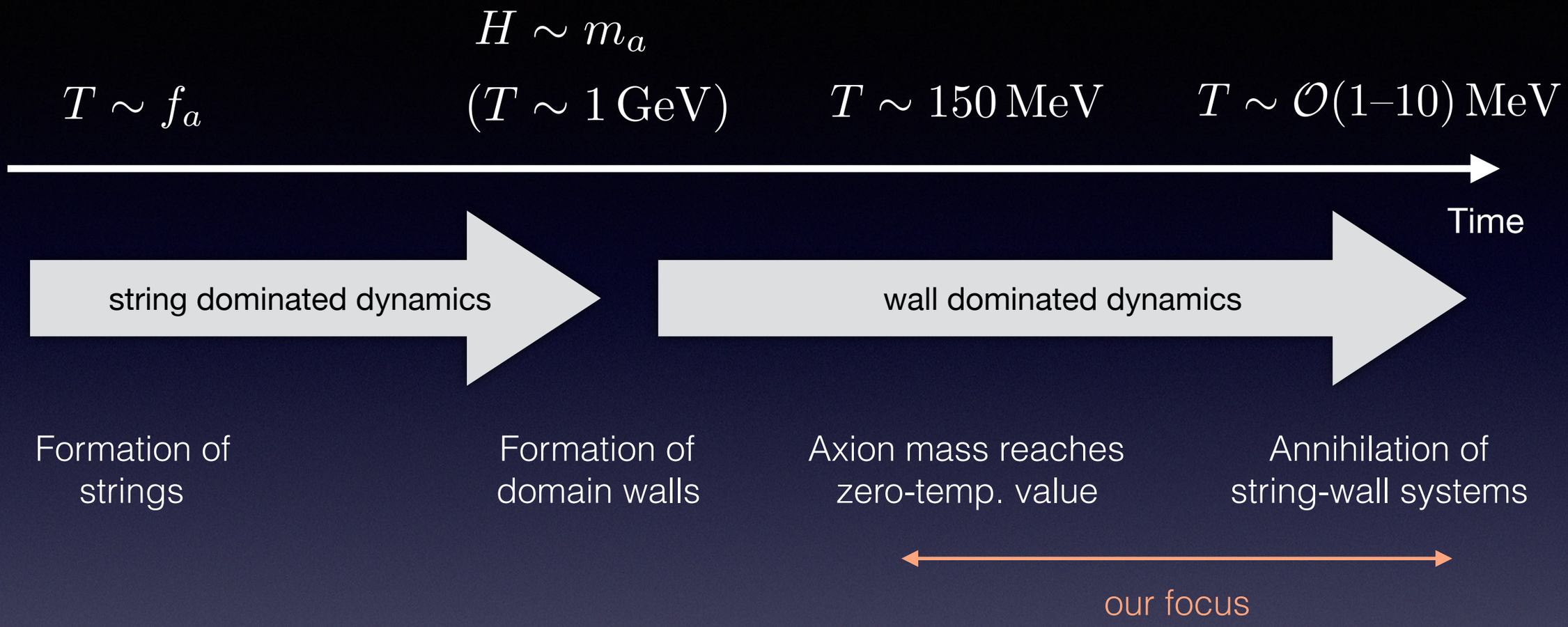
$$V(\phi) = \lambda\left(|\phi|^2 - \frac{v_{\rm PQ}^2}{2}\right)^2 + \frac{m_a^2 v_{\rm PQ}^2}{N_{\rm DW}^2}\left(1 - \frac{|\phi|}{v_{\rm PQ}}\cos\left(N_{\rm DW}\frac{a}{v_{\rm PQ}}\right)\right) \quad (+\Delta V_{\rm bias}(\phi))$$

In the numerical studies, we follow the evolution of two real scalar fields ϕ_1 and ϕ_2 which are defined by $\Phi = \phi_1 + i\phi_2$. We normalize all dimensionful quantities in the unit of η, i.e. $\eta = 1$. With this normalization, the equations of motion for two scalar fields in the FRW background become

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 - \frac{\nabla^2}{R^2(t)}\phi_1 = -\lambda\phi_1(\phi_1^2 + \phi_2^2 - 1) + \frac{m^2}{N_{\rm DW}^2}(\cos\theta\cos N_{\rm DW}\theta + N_{\rm DW}\sin\theta\sin N_{\rm DW}\theta), \quad \text{(A.2)}$$

$$\ddot{\phi}_2 + 3H\dot{\phi}_2 - \frac{\nabla^2}{R^2(t)}\phi_2 = -\lambda\phi_2(\phi_1^2 + \phi_2^2 - 1) + \frac{m^2}{N_{\rm DW}^2}(\sin\theta\cos N_{\rm DW}\theta - N_{\rm DW}\cos\theta\sin N_{\rm DW}\theta). \quad \text{(A.3)}$$

Simulation of $N_{DW} > 1$ domain walls

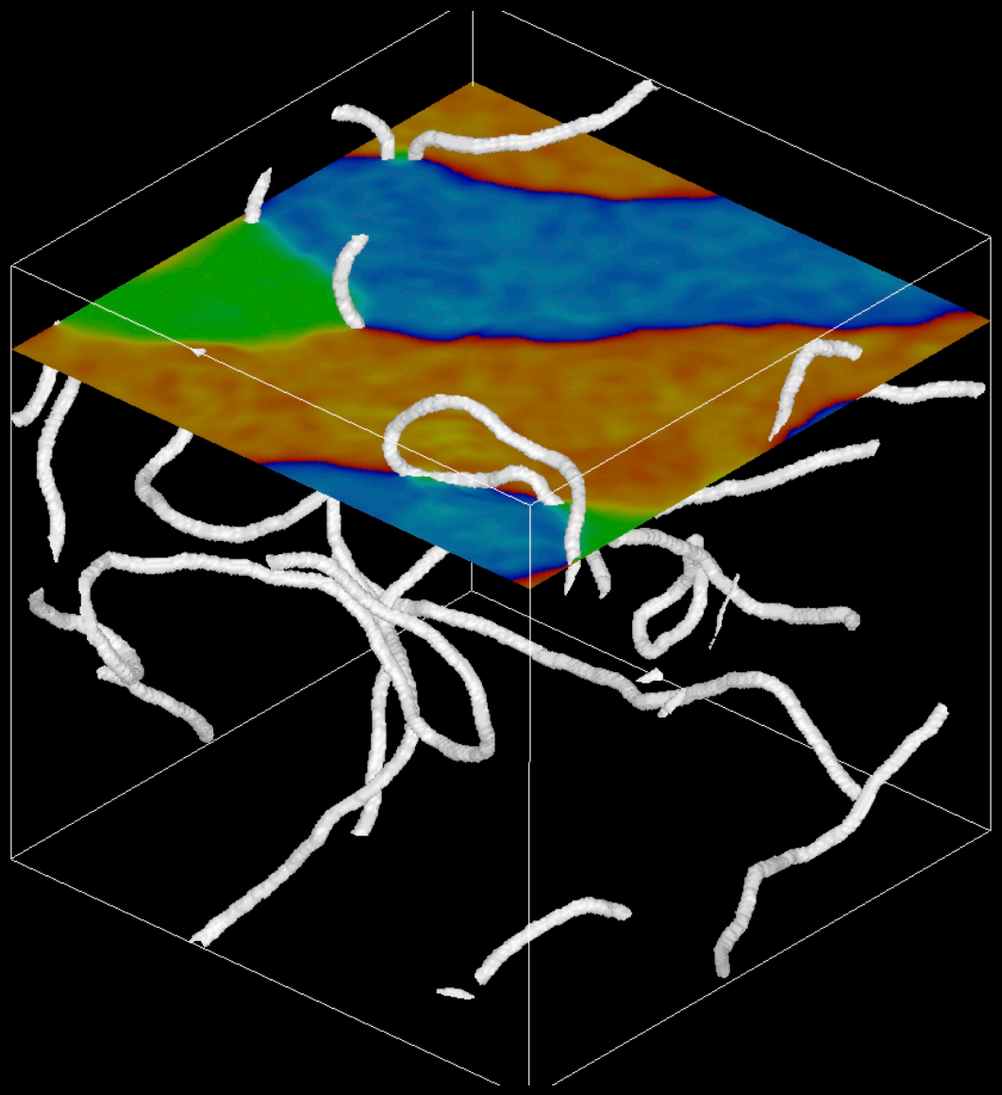


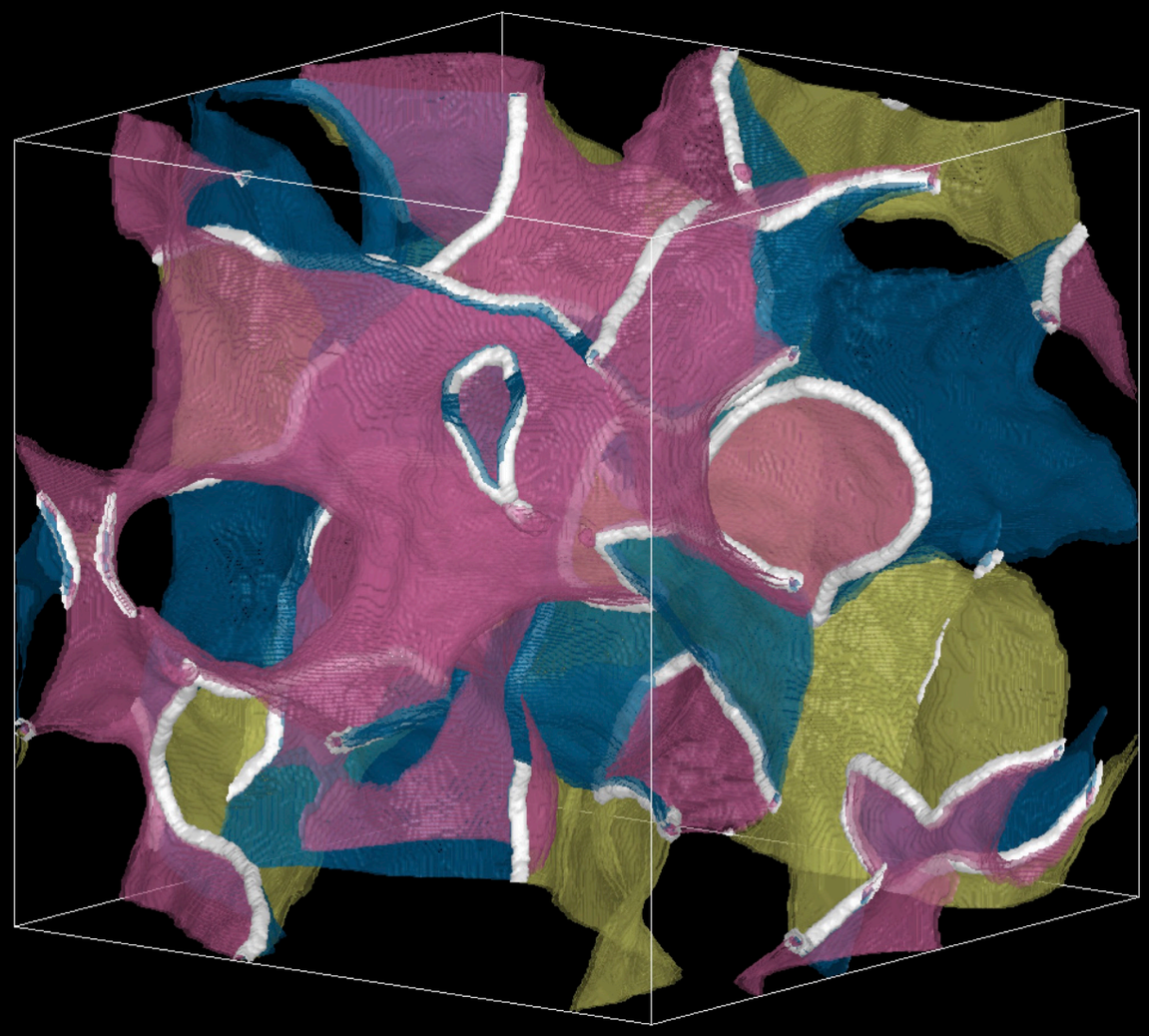
- We used a constant m_a, aiming to produce (and collapse) domain walls in the scaling regime.
- We need $m_a \gg H$ to see the formation of domain walls, but must have $m_a \ll m_s = \sqrt{2\lambda} v_{\mathrm{PQ}}$ in order to avoid the “domain wall instability”.

Simulation of $N_{DW} > 1$ domain walls

[Hiramatsu, Kawasaki, KS and Sekiguchi (2013)]
[Kawasaki, KS and Sekiguchi (2015)]

$$N_{\rm DW} = 3,\ m_a/v_{\rm PQ} = 0.1,\ \text{box size} = 512^3$$



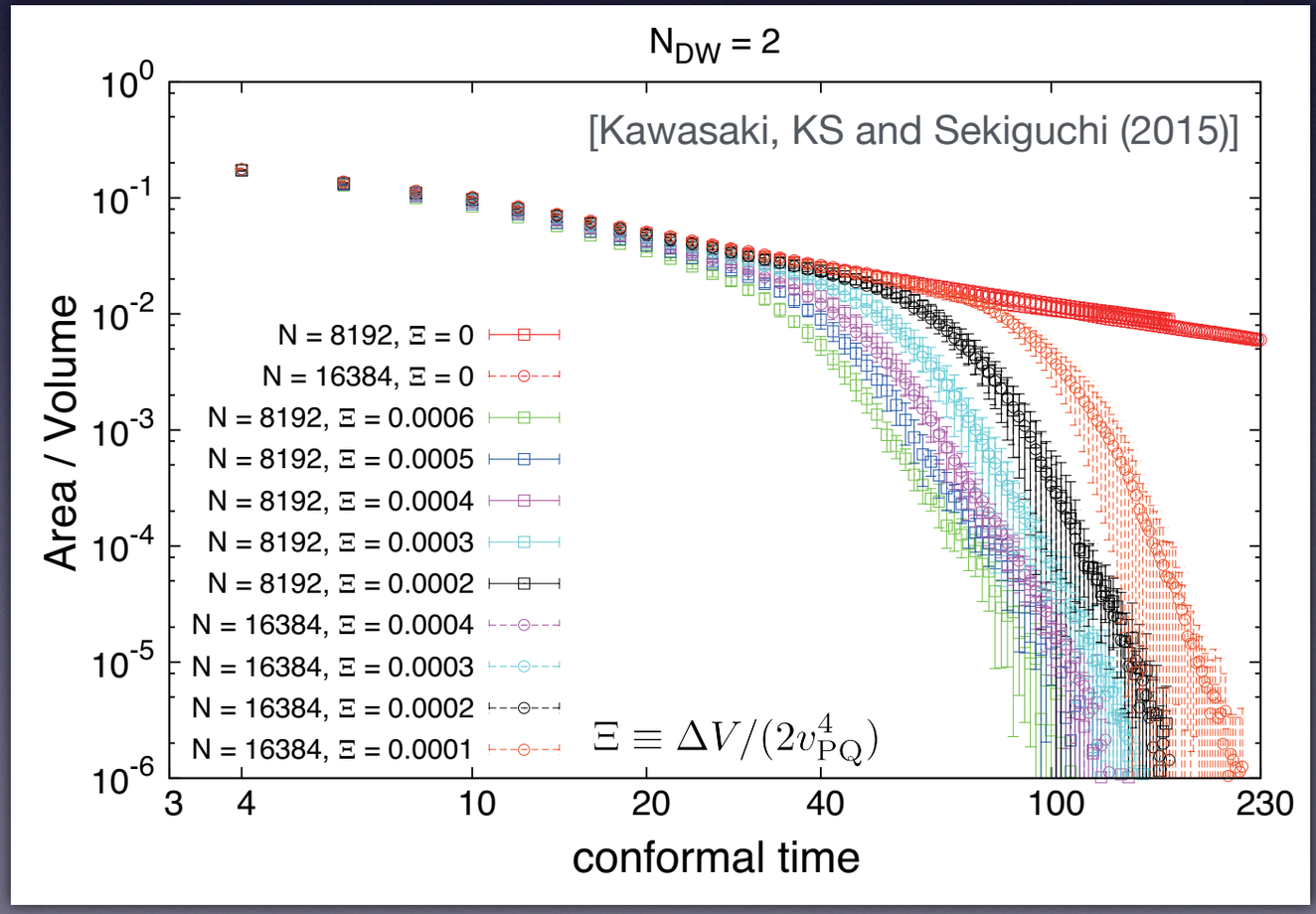


Simulation with the bias term

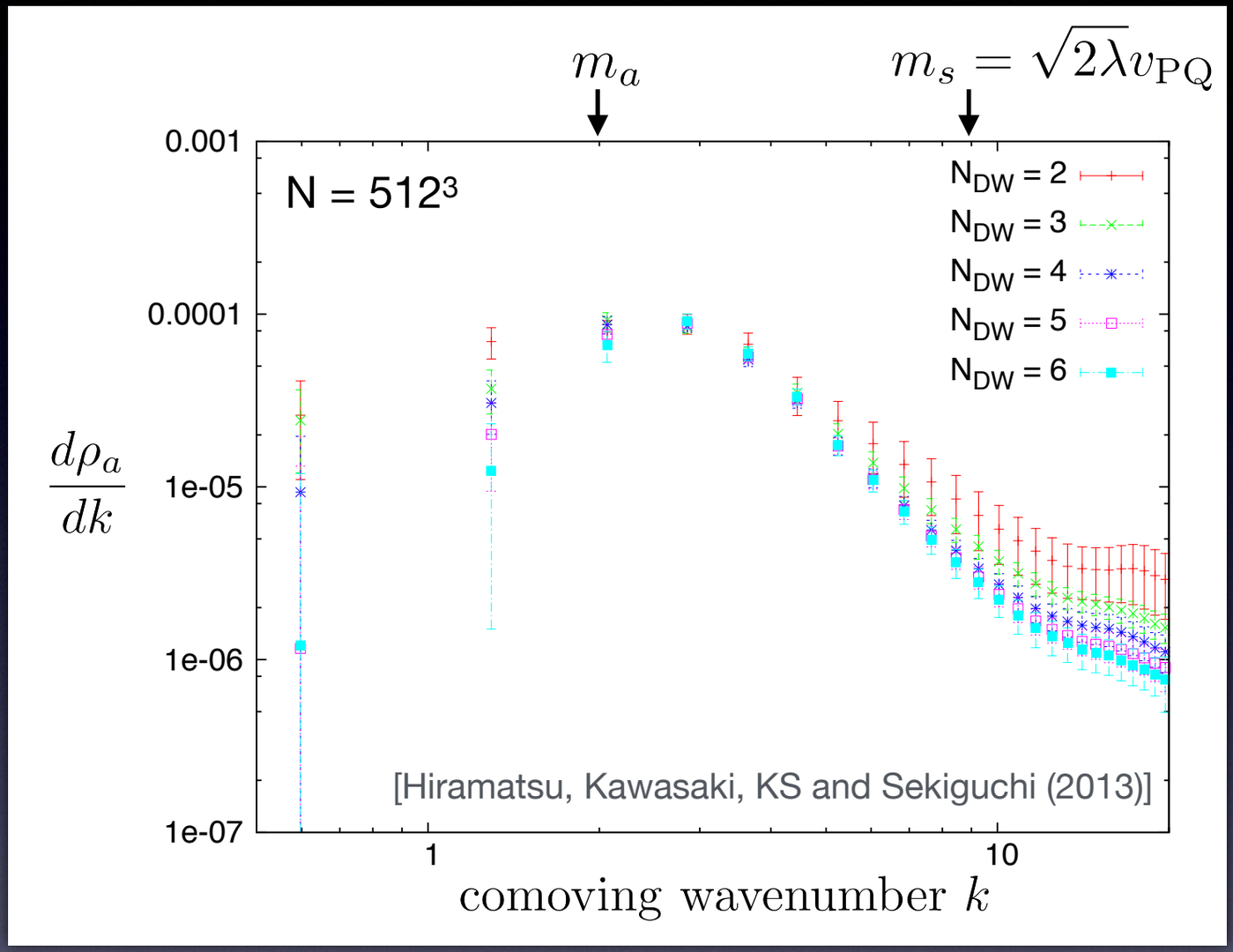
- In order to avoid initial biased distribution of vacua, we require

$$\Delta V / m_a^2 f_a^2 < \mathcal{O}(0.1)$$

- Larger dynamical ranges are required. ➡ 2D (8192^2, 16384^2)
- Walls obey the scaling solution if $\Delta V = 0$
  and decay within the expected time scale if $\Delta V \neq 0$.



Spectrum of radiated axions



- Peak at k ~ axion mass.
- Feature at high k might be contaminated from strings,
  (here we only have $\rho_{\rm str}/\rho_{\rm wall} \sim 0.1$)
  which would be suppressed in the realistic case ($\rho_{\rm str}/\rho_{\rm wall} \sim 10^{-8}$) ?

Estimate of the relic DM abundance

Axion number is frozen at $t \sim t_{\rm dec} \sim \sigma/\Delta V$, until which domain walls produce axions with mean energy $\langle E_a \rangle$,

$$n_a(\text{today}) \approx \left(\frac{R_{\rm dec}}{R_{\rm today}}\right)^3 \frac{\rho_{\rm wall}(t_{\rm dec})}{\langle E_a \rangle}$$

From simulations

$$\rho_{\rm wall}(t) \sim \frac{\sigma}{t}, \quad \langle E_a \rangle \sim m_a$$

Hence

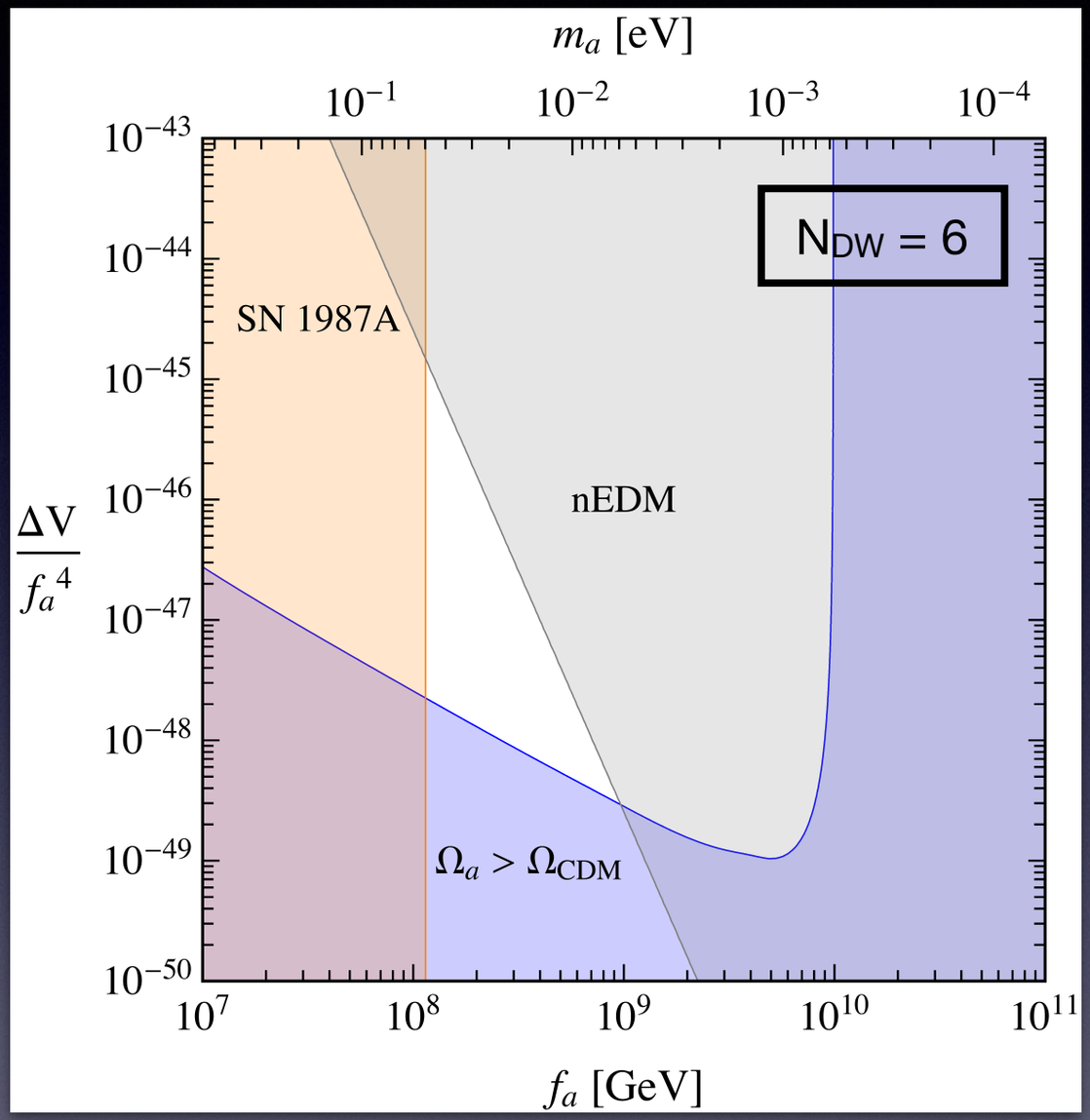
$$\Omega_a h^2 \approx 2 \times 10^{-2}\, C(N_{\rm DW}) \left(\frac{\Delta V/f_a^4}{10^{-48}}\right)^{-1/2} \left(\frac{f_a}{10^9\,\text{GeV}}\right)^{-1/2}$$

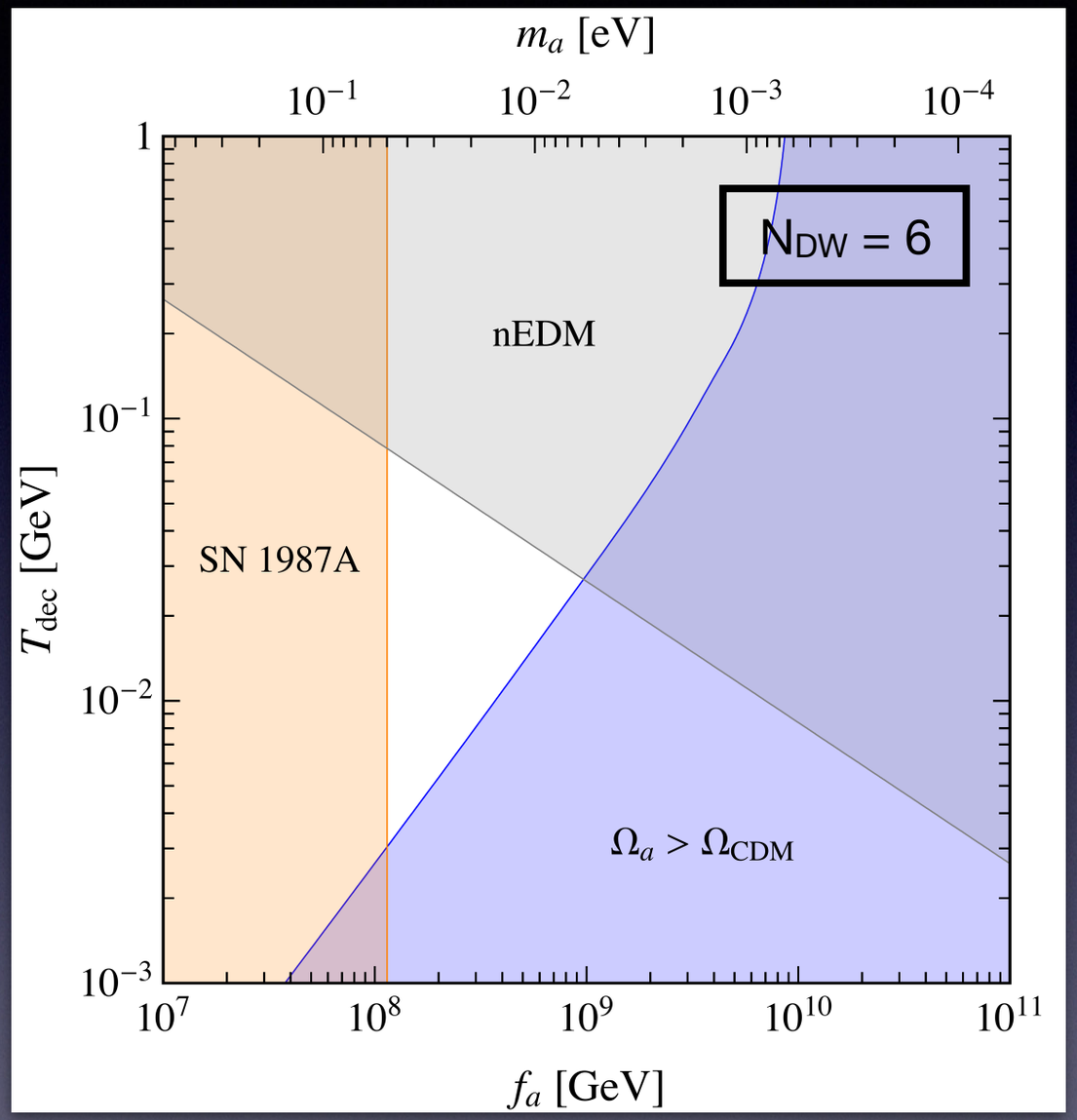
$C(N_{\rm DW})$: value of $\mathcal{O}(1)$ determined from simulations, slightly depend on $N_{\rm DW}$

Note:

For sufficiently large f_a , we have to take account of the contribution from strings. (uncertainty?) [see talks by G. Villadoro, M. Gorghetto, G. D. Moore]

Constraints

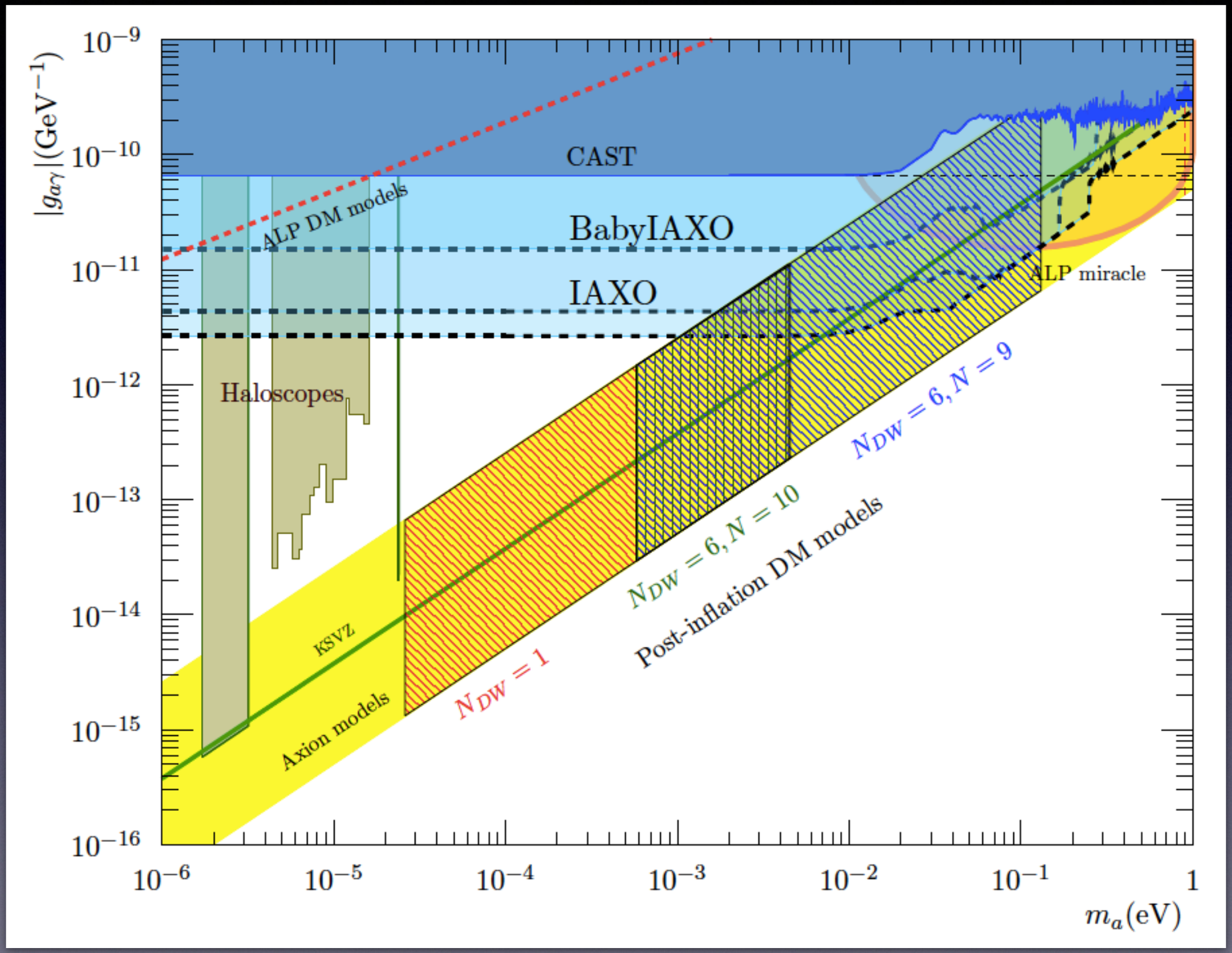




Loophole appears at $\Delta V/f_a^4 \sim 10^{-49}$–$10^{-45}$ and $m_a \sim \mathcal{O}(1\text{–}10)\,\mathrm{meV}$.

Heavier axion dark matter ?

Helioscope searches



[E. Armengaud et al., *Physics potential of the International Axion Observatory (IAXO)* (2019)]

Gravitational waves from domain walls

[Vilenkin (1981); Preskill, Trivedi, Wilczek and Wise (1991); Gleiser and Roberts (1998)]

- Gravitational waves (GWs) are expected to be produced from domain walls until they are annihilated at $T \simeq T_{\rm dec}$.
- Typical frequency and amplitude of GWs: [Hiramatsu, Kawasaki and KS (2014)]

$$f \sim \left(\frac{R(t_{\rm dec})}{R(t_0)}\right) H(t_{\rm dec}) \sim 10^{-9}\,{\rm Hz} \left(\frac{T_{\rm dec}}{10\,{\rm MeV}}\right)$$

$$\rho_{\rm gw}(t_{\rm dec}) \sim G\sigma^2$$

$$\Omega_{\rm gw}h^2(t_0) \sim 3\times 10^{-18} \left(\frac{\sigma}{1\,{\rm TeV}^3}\right)^2 \left(\frac{T_{\rm dec}}{10\,{\rm MeV}}\right)^{-4}$$

$$\sim 8\times 10^{-23} \left(\frac{f_a}{10^9\,{\rm GeV}}\right)^2 \left(\frac{T_{\rm dec}}{10\,{\rm MeV}}\right)^{-4}$$

Amplitude is too small to observe. [Hiramatsu, Kawasaki, KS and Sekiguchi (2013)]
(otherwise cold axions are overproduced)

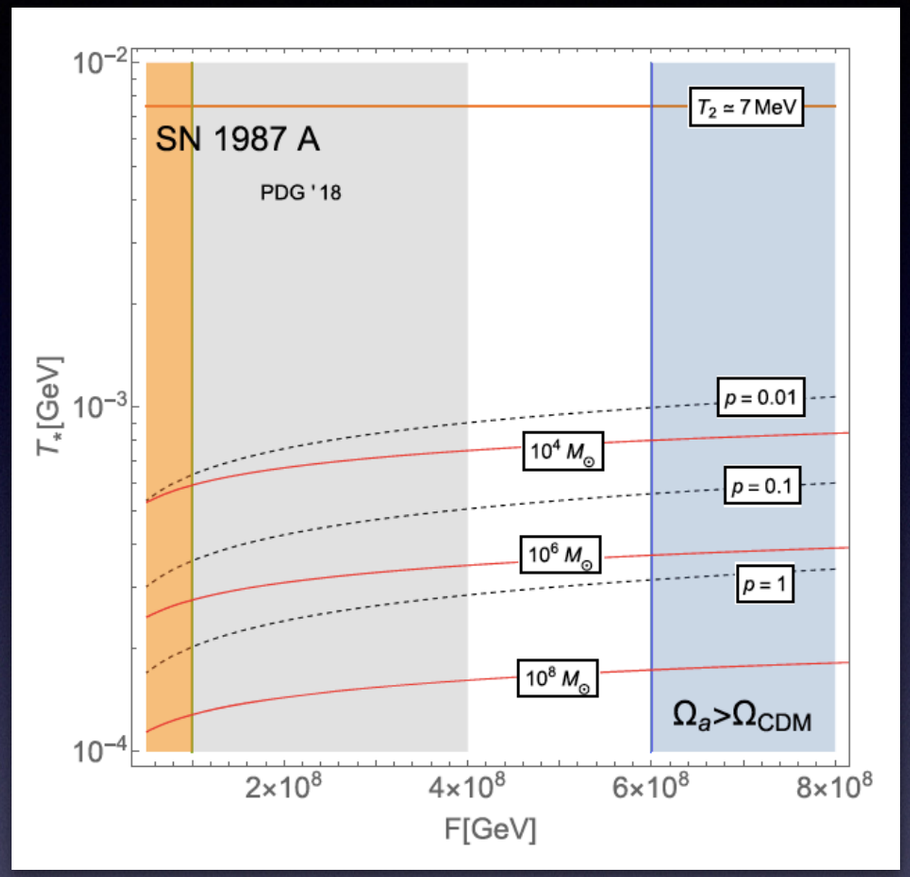
Primordial black holes

[Ferrer, Masso, Panico, Pujolas and Rompineve (2019)]

- Closed wall with size $R_\star$ starts to collapse when $R_\star \sim H_\star^{-1}$.
- The total collapsing mass:

$$M_\star \sim 4\pi H_\star^{-2}\sigma + \underline{\frac{4}{3}\pi H_\star^{-3}\Delta V}$$

dominant if $T_\star < T_{\rm dec}$



- "Figure of merit" for PBH formation

$$p = \frac{R_{\rm Sch}}{R_\star} \sim \frac{2GM_\star}{H_\star^{-1}} \sim \frac{H_\star^{-2}\Delta V}{3M_{\rm Pl}^2} \propto T_\star^{-4}$$

- Possibility for the formation of PBHs with

$$M_\star \sim 10^4\text{–}10^7\, M_\odot, \quad f = \Omega_{\rm PBH}/\Omega_a \sim 10^{-6}$$

Explanation of the supermassive BHs?

Miniclusters?

Very naive picture:

DM density could be inhomogeneous on a scale $H(T_{\rm dec})^{-1}$ corresponding to the horizon at the domain wall decay rather than the QCD horizon $H(T_{\rm QCD})^{-1}$.

Comoving horizon size

$$L = \frac{1}{R_{\rm dec} H_{\rm dec}} = 8.5\,{\rm pc} \left(\frac{T_{\rm dec}}{10\,{\rm MeV}}\right)^{-1}$$

More massive and larger miniclusters??

$$M_0 \sim 10^{-5}\,M_\odot \left(\frac{\Omega_a h^2}{0.12}\right) \left(\frac{T_{\rm dec}}{10\,{\rm MeV}}\right)^{-3}$$

$$R \sim 10^{11}\,{\rm km} \left(\frac{T_{\rm dec}}{10\,{\rm MeV}}\right)^{-1}$$

Need detailed studies to address quantitative issues…

Conclusion

- Several well-motivated models predict $N_{DW} > 1$.
- Serious domain wall problem, but there is a loophole.
- Non-trivial consequences for astrophysics:
  - Heavier axion dark matter mass.
  - PBHs, axion miniclusters, …
- Future works for simulations:
  - More realistic initial conditions (e.g. temperature dependent axion mass, denser strings).
  - Larger dynamical ranges to confirm (or disprove) the scaling behavior.
  - Study of superlarge domain walls to determine PBH fraction.
  - Investigation of minicluster seeds.